UNICIV REPORT 211

UNICIV R-211 R.I. GILBERT

CONTENTS

1. INTRODUCTION

The design of a reinforced concrete slab is complicated by the difficulties involved in estimating the service load behaviour. The designer is first confronted with the problem of selecting a suitable slab thickness. A reasonable first estimate is desirable since, in many cases, the slab self-weight is a large proportion of the total service load. Strength considerations alone generally result in the selection of a slab depth which leads to in-service problems, in particular, excessive deflections. The relatively low steel percentages commonly used in slabs are evidence of either the conscious or the unconscious consideration of serviceability at the design stage.

In AS 1480-1974, two alternative procedures are proposed for the control of deflections. Deflections may be calculated directly and the magnitudes compared with specified deflection limits. Although simple, semi-empirical methods have been developed to predict the deflections of reinforced concrete beams, the same is not true for two-way slab systems. The three dimensional nature of the problem, the less well defined influence of cracking and tension stiffening, and the development of biaxial creep and shrinkage strains create additional difficulties. Practising engineers have no reliable method for estimating either short-term or long-term slab deflections and the code gives no guidance.

Alternatively, AS 1480-1974 suggests that deflections need not be calculated if the adopted span to effective depth ratio (L/d) is less than a maximum specified value. This approach is simple and therefore ideal for use in routine design. However, it has been shown (6, 9, 12) that the maximum span to depth ratios specified in AS 1480-1974 are not adequate for controlling deflections. This inadequacy has been recognised in the initial draft of the proposed Unified Concrete Code of the Standards Association of Australia. Recently Rangan (8) proposed a simple expression for allowable span to depth ratios for reinforced concrete beams which appears to ensure adequate deflection control. Rangan's expression was developed from the deflection computation procedure proposed by Branson (2,3) which has been widely accepted (1,10) and agrees well with test data.

In this paper, Rangan's expression for the maximum allowable span to depth ratios for reinforced concrete beams is extended to include the cases of two-way edge-supported slabs, flat slabs and flat plates. This extension stems from an earlier investigation of span to depth ratios for slabs (6) and is based on data obtained from an extensive series of parametric computer experiments with reinforced concrete slabs (4,5). The proposed extension

/2.

of Rangan's expression provides a rational procedure for obtaining a maximum allowable span to depth ratio for the entire range of reinforced concrete flexural members. The procedure is at once simple and flexible and provides the designer with a reliable and sure means for the control of deflections.

2. ALLOWABLE SPAN TO DEPTH RATIOS FOR BEAMS

2.1 Rangan's Expression

Using an elastic analysis and a simplification of Branson's equation (2) for the effective moment of inertia of a cracked section, Rangan (8) proposed the following expression (with some notation changes) for the maximum allowable span to depth ratio of a reinforced concrete beam:

$$L/d \leqq \lambda_1 \ \lambda_2 \ \left[\frac{\Delta}{L} \left(\frac{\alpha \ b_{ef} \ E_c}{w_v + c \ w_s}\right)\right]^{0.33} \quad (1)$$

where λ_1 is a factor which depends on the support conditions and may be taken as 1.0 for a simply-supported member

1.3 for an exterior span of a continuous member

1.5 for an interior span of a continuous member

and 0.3 for a cantilever.

λ_2 is a factor for flanged beams and is similar to the multipliers recommended in Table 10.1.4(4) of AS 1480-1974 (10). λ_2 varies from 0.8 for narrow webbed flanged beams to 1.0 for rectangular sections. α accounts for the effect of the reinforcement ratio, p, and the modular ratio, n, on the effective moment of inertia of the beam after cracking and

$$\begin{aligned} \alpha &= 15\sqrt{p \ n} \leqq 8.0 \quad &\text{for } p \ n > 0.045 \\ \alpha &= 1/7 \ p \ n \leqq 5.0 \quad &\text{for } p \ n \leqq 0.045 \end{aligned} \quad (2)$$

b_{ef} is the effective width of the compressive face at the midspan region (or at the support of a cantilever).

L is the effective span.

E_c is the elastic modulus of concrete.

w_s is the sustained load per unit length.

w_v is the variable load per unit length.

If the member is NOT supporting non-structural elements likely to be damaged by excessive deflection

Δ is the maximum permissible total deflection (say L/240)

c is equal to 1 + F

For members supporting non-structural elements likely to be damaged by excessive deflection -

Δ is the maximum permissible incremental deflection (or that part of the total deflection which occurs after the attachments of non-structural elements, say L/500 or L/1000);

c is equal to F

and F is the long-term deflection multiplication factor (10)

$$F = 2.0 - 1.2\, A_{sc}/A_{st} \geqq 0.6 \tag{3}$$

2.2 Discussion

In the development of equation 1, the effective moment of inertia of the cracked beam section under the maximum in-service moment, M, was chosen to provide close agreement with Branson's well-known equation for I_e, namely

$$I_e = (M_c/M)^3\, [I_g - I_{cr}] + I_{cr} \tag{4}$$

The cracking moment, M_c, is usually deemed to occur when the extreme tensile fibre of the gross section reaches a limiting stress of $0.62\sqrt{F'_c}$ (10).

It is argued, with justification, that the use of equation 1 for limiting the span to depth ratio of a beam is as reliable as limiting the deflection of a beam calculated using equation 4. It is perhaps worthwhile here to briefly comment on the applicability of equation 4.

Branson's equation was developed from a statistical study of 54 short-term test specimens which all were loaded to at least twice the cracking moment and which all had reinforcement ratios greater than 0.006. It is of interest to note that most reinforced concrete slabs fall completely outside the limits of Branson's study.

For lightly loaded members with low reinforcement ratios, such as most slabs, the maximum in-service moment may be little more than the cracking moment. In such cases, equation 4 suggests that little loss of stiffness due to cracking occurs. However, cracking may occur at load levels well below that assumed by Branson due to tensile stresses induced by shrinkage and by temperature changes. In fact, shrinkage induced cracking in slabs can reduce the flexural stiffness even before the formwork is removed and the slab is first loaded.

The use of equation 4, together with the long-term deflection multiplier (equation 3), can therefore greatly underestimate slab deflections. Rangan recognised this inadequacy and proposed an upper limit on the cracked stiffness of beams with low reinforcement ratios (equation 2). The accuracy of this expedient will be examined in Section 3.3.

3. ALLOWABLE SPAN TO DEPTH RATIOS FOR SLABS

3.1 Introduction

To extend Rangan's expression (equation 1) to include the case of two-way slabs, several difficulties need to be overcome. Apart from the possible inaccuracy of equation 4 for members with low reinforcement ratios, the need to model plate action rather than beam action becomes paramount. The two-way nature of the load dispersion and the stiffness of the orthotropic reinforced concrete plate must be accounted for quantitatively.

Previous attempts to account for these factors and to develop simplified, design oriented procedures for the control of slab deflections have been hampered by a lack of suitable experimental data. Little research effort has been expended on experimental investigations of the in-service behaviour of slabs. It is only in recent years that sophisticated theoretical models have been developed as research tools to generate the extensive data necessary for the calibration of simplified models and procedures.

In the following sections, the extension of Rangan's expression to include slabs is calibrated using the results obtained from a series of parametric computer experiments using one such theoretical research model.

3.2 Computer Experimentation

The computer experiments were performed using a layered, compatible, plate bending, finite element model which was developed to study the long-term service-load behaviour of reinforced and prestressed concrete slabs (4). This slab simulation model accounts for the various sources of material non-linearity, such as cracking, tension stiffening, creep and shrinkage, and has been shown to accurately predict both the instantaneous and time-dependent behaviour of a variety of concrete slabs.

Using the finite element model, several reinforced concrete slab types were analysed, including one-way slabs, two-way edge-supported slabs, flat slabs and flat plates. The aim of this experimentation was to develop a set of limiting L/d ratios which provides adequate control of in-service deflections for each slab type. The advantage of this approach is that results are obtained from a model which attacks the problem at a 'constitutive relationship' level and which has been shown to agree well with macroscopic measurements of structural behaviour. No use is made of empirical estimates of stiffness or conclusions drawn from experimental testing of statically determinate concrete beams. A description of the computer investigation has been reported elsewhere (4,5).

To allow the results of the investigation to be of use to practising engineers, a semi-empirical expression for the maximum allowable L/d ratio, which provided close agreement with the experimental results for each slab type, was previously developed (6). Whilst this expression has been shown to provide adequate deflection control for slabs, it is restricted by the scope of the computer investigation and cannot be extended to cover the entire range of reinforced concrete flexural members.

Following the development of equation 1, it was decided to re-examine the computer results in an attempt to develop a 'slab system factor' which could be used to satisfactorily modify Rangan's expression.

3.3 One-Way Slabs

In Table 1, a comparison is made between the maximum permissible L/d ratios determined experimentally on the computer and those obtained using equation 1 for a number of simply-supported one-way slabs. The maximum total deflection was limited to L/250 and the characteristic compressive strength of the concrete and the yield stress of the steel were F'_c = 25 MPa and f_{sy} = 410 MPa, respectively.

The L/d ratio obtained using equation 1 was determined by considering an equivalent one metre wide beam with the same span and support conditions as the one-way slab. The properties of the slab, which had a maximum total deflection of L/250, were determined using successive iterations on the computer and then fed into equation 1 to compare the two approaches. In each case the quantity of tensile reinforcement was that required to satisfy the design objective of adequate strength.

TABLE 1

COMPARISON OF MAXIMUM PERMISSIBLE L/d RATIOS FOR SIMPLY-SUPPORTED ONE-WAY SLABS

Slab No.	Span (m)	w_v (kN/m^2)	P ($=A_{st}/bd$)	Maximum Permissible L/d	
				Experimental	Equation 1
1	3.0	1.0	0.0029	33.0	31.8
2	5.0	1.0	0.0030	28.0	27.6
3	7.0	1.0	0.0031	23.6	24.2
4	3.0	3.0	0.0036	29.8	28.7
5	5.0	3.0	0.0036	26.2	25.4
6	7.0	3.0	0.0036	22.8	22.8
7	3.0	5.0	0.0042	27.5	26.0
8	5.0	5.0	0.0040	24.4	23.6
9	7.0	5.0	0.0040	21.7	21.4

Note: The sustained load for all slabs was $w_s = 1.0\ kN/m^2$ + self weight.

In each case, the slab reinforcement ratio is low. This indicates that stiffness rather than strength is the main design concern.

Despite great differences in the methods of development, agreement between the two approaches is excellent and Rangan's upper limit on the cracked stiffness of members with low reinforcement ratios (equation 2) appears to be well calibrated.

3.4 Edge-Supported Two-Way Slabs

In addition to the factors which affect beam and one-way slab deflections, the deflection of a two-way, edge-supported, rectangular slab panel depends on the boundary conditions on all four sides and, importantly, on the aspect ratio, i.e. the ratio of the longer to the shorter panel dimension (l_y/l_x). The load on the slab is resisted not only by orthogonal bending moments but also by twisting moments.

In order to apply equation 1 to two-way slabs, it is necessary to devise a simple yet reasonably accurate 'equivalent beam' which suffers the same overall deflection as the uniformly loaded slab. Figure 1 shows a pair of one metre wide beams at right angles through the centre of the slab panel, similar to those first suggested by Marsh (7) in 1904. The support of each beam has the same conditions of continuity as the corresponding edge of the slab. By enforcing the requirement that the midspan deflection of each beam is equal, the fraction of the uniformly distributed load carried by the beam strip in the short span direction may be readily calculated.

The shorter span beam in Figure 1 has been selected as the 'equivalent beam' and the uniformly distributed load per metre resisted by the beam is calculated as indicated above. If the two orthogonal beams have similar support conditions, the fraction, k, of the total load carried by the equivalent beam may be approximated by the well known expression -

$$k = \frac{l_y^4}{l_x^4 + l_y^4} \qquad (5)$$

Of course, this result is very approximate because it neglects the twisting moments in the slab and assumes that the flexural rigidity of each beam is similar.

To overcome these sources of inaccuracy and to provide close agreement with the computer predictions, a 'slab system factor', λ_3, has been introduced as an additional multiplication factor in equation 1. Values of λ_3 for two way edge-supported slabs depend on the aspect ratio of the slab and are given in Table 2.

3.5 Flat Slabs and Flat Plates

Figure 2 shows a typical portion of a flat slab. The 'equivalent beam' adopted for use with Rangan's expression is a one metre wide beam located on the column line in the long-span direction. The reinforcement at any point along the equivalent beam is the average of the appropriate column and middle strip reinforcement quantities per metre width. Since the total load is carried in both directions by the slab, the fraction k in a flat slab is unity.

The slab system factor, λ_3, for flat slabs with and without drop panels is given in Table 2 and, once again, was calibrated using the data obtained from the complex computer based slab simulation model.

TABLE II.

SLAB SYSTEMS FACTOR, λ_3.

Slab Type	Aspect Ratio			
	1.0	1.25	1.50	2.0
One-Way Slab:	1.00	1.00	1.00	1.00
Two-Way Edge-Supported Slab:	1.23	1.15	1.09	1.04
Flat Slab*:				
With Drop Panels:	0.94	1.00	1.03	1.08
Without Drop Panels:	0.86	0.91	0.95	1.00

*Note: If an exterior panel of a flat slab has stiff spandrel beams perpendicular to the equivalent beam, the factor λ_3 may be increased by an additional factor of 1.03. Drop Panels are assumed to satisfy the requirements of AS 1480 (10) and are at least 33 percent thicker than the adjacent slab.

3.6 Proposed Procedure

The maximum allowable span to depth ratio for a reinforced concrete slab may be calculated by applying equation 6 to the appropriate 'equivalent beam' as defined in Sections 3.3, 3.4 and 3.5

$$L/d \leqq \lambda_1 \ \lambda_2 \ \lambda_3 \ \left[\frac{\Delta}{L} \left(\frac{\alpha \ b_{ef} \ E_c}{k(w_v + c \ w_s)}\right)\right]^{0.33} \tag{6}$$

where λ_1 is the factor which depends on the support conditions of the equivalent beam and is defined in Section 2.1.

λ_3 is the slab system factor (Table 2).

k is the fraction of the total uniformily distributed load carried by the equivalent beam and is unity for one-way slabs and flat slabs.

b_{ef} is the effective width of the equivalent beam (one metre)

w_s and w_v are the sustained and variable loads, respectively, carried by the equivalent beam per unit length and are numerically equal to the sustained and variable slab loads per square metre.

All other notation is as defined in section 2.1.

4. APPLICATION

4.1 Worked Example

A two-way edge-supported slab is discontinuous on one long edge and continuous on the remaining three edges. The design data for the rectangular panel is given below:

$l_y = 7.5$ m; $l_x = 5.0$ m; $w_v = 3.0$ kPa; $w_s = 7.0$ kPa;

$F'_c = 25$ MPa; $f_{sy} = 410$ MPa; $E_c = 25\ 000$ MPa;

and the maximum permissible total deflection is L/250 (or 20 mm in this case).

The aim is to find the minimum required slab thickness.

It is first necessary to determine what portion of the total load, w, is carried by the equivalent beam, which in this case is continuous at one support only. The approximate procedure outlined in Section 3.4 is used here.

The midspan deflection, Δ, of the equivalent beam under the load kw is approximated by

$$\Delta = \frac{2}{384} \frac{kw\ l_x^4}{EI} \tag{7}$$

The beam through the slab centre at right angles to the equivalent beam is continuous at both ends and suffers a midspan deflection of

$$\Delta = \frac{1}{384} \frac{(1-k)w\ l_y^4}{E\ I} \tag{8}$$

The fraction k is obtained by equating the midspan deflection of each beam. Therefore

$$2\ kw\ l_x^4 = (1-k)w\ l_y^4$$

and $$k = \frac{l_y^4}{2\ l_x^4 + l_y^4} = 0.72$$

To obtain an approximate minimum slab depth, an estimate of the average reinforcement ratio is required. A reasonable first estimate is $p = 0.004$ and from equation 2, $\alpha = 4.46$. The other parameters required for substitution in equation 6 are as follows:

$\lambda_1 = 1.3$; $\lambda_2 = 1.0$; $\lambda_3 = 1.09$; $\Delta = 20$ mm; $c = 3.0$; $L = 5000$ mm;

$b_{ef} = 1000$ mm; $w_v = 3.0$ kN/m; and $w_s = 7.0$ kN/m.

By substituting into equation 6, the minimum L/d ratio is found to be 40.5 and the corresponding minimum effective depth is 5000/40.5 = 124 mm. Using 12 mm diameter reinforcing bars and 20 mm minimum cover to the steel in the short-span direction results in a minimum slab thickness of 150 mm.

With this size slab, the reinforcement in the short-span direction required to satisfy the design objective of adequate strength is approximately $p = 0.0036$ in the positive moment region and $p = 0.0048$ over the continuous support. The average value for p assumed in the calculations above is considered acceptable and no refining iterations are necessary.

4.2 Comparison with Test Data

No well documented, laboratory controlled long-term tests of two-way reinforced concrete slabs exist. However, field measurements on several in-service slabs were presented by Taylor (11) and form a suitable set of data for comparison with predictions made using equation 6.

Case 1:

An internal panel of a flat plate roof slab, with plan dimensions of 6.35 m by 5.08 m measured from centre to centre of columns, was analysed. The slab was 203 mm thick with an effective depth to the steel in the long span direction of 173 mm. The actual L/d ratio was therefore 36.7. The sustained load, including the self weight, was 5.27 kPa, construction loads were negligible, and the roof was non-trafficable. The variable load was therefore taken to be zero.

The average reinforcement ratio in the 'equivalent beam' and the other parameters required for substitution into equation 6 are listed below and are obtained from the reinforcement quantities and material properties reported by Taylor (11).

$\lambda_1 = 1.5$; $\lambda_2 = 1.0$; $\lambda_3 = 0.91$; $p = 0.0049$; $\alpha = 3.70$; $E_c = 25\ 160$ MPa;

$b_{ef} = 1\ 000$ mm; $w_s = 5.27$ kN/m; $w_v = 0$; $L = 6\ 350$ mm; $k = 1$; and

F is taken as 2.0.

The total midpanel deflection measured by Taylor was $\Delta = 19$ mm after $2\frac{1}{2}$ years. With this data, equation 6 suggests that the L/d is

$$L/d = 1.5 \times 1.0 \times 0.91 \left[\frac{19.0}{6350} \frac{3.70 \times 1000 \times 25160}{3 \times 5.27}\right]^{0\cdot 33}$$

$$= 34.4$$

The incremental deflection, which occurred after the initial or short-term deflection, was 15.5 and if this is used in equation 6, the required L/d is

$$L/d = 1.5 \times 1.0 \times 0.91 \left[\frac{15.5}{6350} \frac{3.70 \times 1000 \times 25\ 160}{2 \times 5.27}\right]^{0\cdot 33}$$

$$= 36.8$$

Both these predictions are in close agreement with the actual span to depth ratio of the slab.

Case 2:

An internal panel of a flat plate floor in an open air carpark, which was constructed in 1965 and monitored by Taylor (11) for a period of $3\frac{1}{2}$ years, was analysed. The slab was 241 mm thick with 2440 x 2440 x 152 mm drop panels over each column and an effective depth to the long span reinforcement at midspan of 212 mm. The panel plan dimensions were 8.61 m by 7.93 m, measured centre to centre of the columns, and the actual L/d ratio was therefore 8610/212 = 40.6. The sustained load was 5.75 kPa and the variable live load was 2.87 kPa.

The reinforcement layout and material properties used in the analysis of the equivalent beam are as detailed by Taylor. It is noted that the slab reinforcement was designed by assuming it to be a flat plate without drop panels. The presence of the drop panels increases the slab stiffness and provides a large reserve of strength. At the same time, the relatively deep drops reduce the reinforcement ratio, p, over the columns.

The parameters required for substitution into equation 6 are

$\lambda_1 = 1.5$; $\lambda_2 = 1.0$; $\lambda_3 = 0.96$; $\alpha = 5.0$; $b_{ef} = 1\ 000$ mm; $E_c = 24\ 500$ MPa; $L = 8\ 610$ mm; $w_s = 5.75$ kN/m; and $w_v = 2.87$ kPa.

The incremental deflection which occurred with time after the removal of the formwork was $\Delta = 18$ mm and, from equation 6, we have

$$L/d = 1.5 \times 1.0 \times 0.96 \left[\frac{18.0}{8610} \frac{5.0 \times 1\ 000 \times 24\ 500}{2.87 + 2.0 \times 5.75}\right]^{0\cdot 33}$$

$$= 36.4$$

This is a little less than the actual L/d ratio. However, the drop panels were deeper than those used in the computer experiments which led to the calibration of the slab system factor, λ_3, (a 62% increase in slab thickness compared with the 33% increase assumed) and a smaller L/d ratio is to be expected.

5. CONCLUSION

A simple, design oriented procedure for the control of deflections in reinforced concrete slab systems has been proposed. A maximum allowable span to depth ratio is calculated using a rational procedure first developed for beams by Rangan (8) and extended to cover the entire range of reinforced concrete flexural members. The procedure was calibrated using data generated by a finite element slab simulation model and has been shown to agree well with field measurements of in-service slabs.

6. REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete", ACI 318-77, American Concrete Institute, Detroit, 1977.
2. Branson, D.E., "Instantaneous and Time-Dependent Deflections of Simple and Continuous Reinforced Concrete Beams", Part 1, Report No. 7, Alabama Highway Research Report, Bureau of Public Roads, August 1963, pp. 1-78.
3. Branson, D.E., "Design Procedures for Computing Deflections", ACI Journal, Proc. Vol. 65, September 1968, pp.730-742.
4. Gilbert, R.I., "Time-Dependent Behaviour of Structural Concrete Slabs", Ph.D.Thesis, University of New South Wales, Australia, March 1979, p.361.
5. Gilbert, R.I., "Computer Experiments with Reinforced Concrete Slabs", Uniciv Report No.R-195, The University of New South Wales, Australia, March 1980.
6. Gilbert, R.I., "Allowable Span to Depth Ratios for Reinforced Concrete Slabs", Proceedings of the Seventh Australasian Conference on the Mechanics of Structures and Materials, University of Western Australia, Perth, May 1980, pp.103-109.
7. Marsh, C.F., "Reinforced Concrete", D. Van Nostrand Co., New York, 1904, p.283.

8. Rangan, B.V., "Maximum Allowable Span/Depth Ratios for Reinforced Concrete Beams", Civ. Engg. Trans., I.E. Aust., Vol. CE 24, No. 4, Nov. 1982, pp.312-317.

9. Rangan, B.V. and Donaghy, T., "Deflection Design of Flexural Concrete Members by Allowable Span/Depth Ratios", Civ. Engg. Trans., I.E. Aust., Vol. CE20, No. 1, 1978, pp.8-12.

10. Standards Association of Australia, "SAA Concrete Structures Code", AS 1480-1974.

11. Taylor, P.J., "The Initial and Long-Term Deflections of Reinforced Concrete Flat Slabs and Plates", M.E. Thesis, The University of New South Wales, Australia, 1971.

12. Walsh, P.F., "Deflection of Reinforced Concrete", Civ. Engg. Trans., I.E. Aust., Vol. CE 19, No. 2, 1977, pp.146-152.

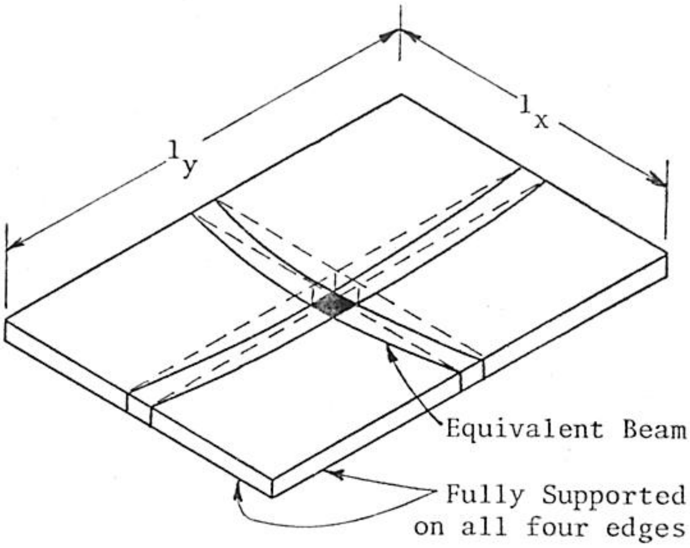

Figure 1 The 'Equivalent Beam' in a Two-Way Edge Supported Slab

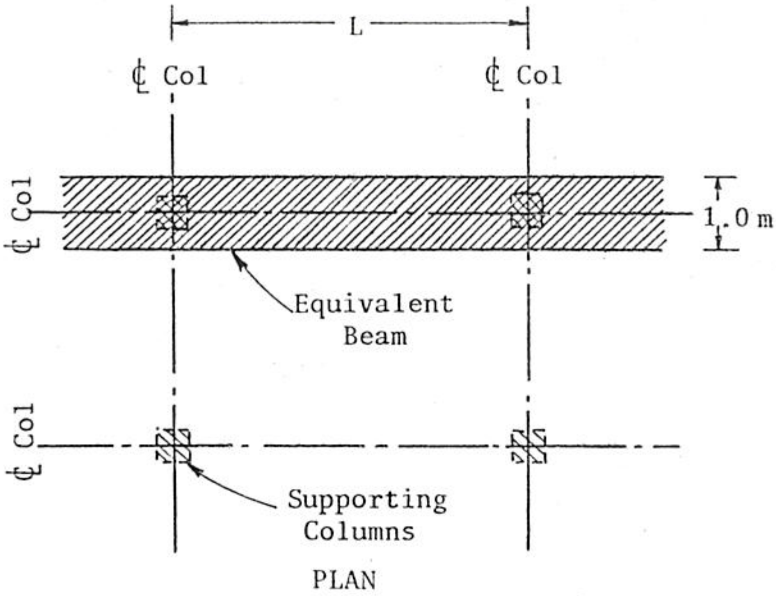

Figure 2 The 'Equivalent Beam' in a Flat Slab or Flat Plate.