

MAXIMUM HYPERGRAPHS WITHOUT REGULAR SUBGRAPHS

JAEHOON KIM¹

*Department of Mathematics, University of Illinois
Urbana, IL, 61801, USA*

e-mail: kim805@illinois.edu

AND

ALEXANDR V. KOSTOCHKA²

*University of Illinois at Urbana–Champaign,
Urbana, IL 61801, USA
Sobolev Institute of Mathematics
Novosibirsk 630090, Russia*

e-mail: kostochk@math.uiuc.edu

Abstract

We show that an n -vertex hypergraph with no r -regular subgraphs has at most $2^{n-1} + r - 2$ edges. We conjecture that if $n > r$, then every n -vertex hypergraph with no r -regular subgraphs having the maximum number of edges contains a full star, that is, 2^{n-1} distinct edges containing a given vertex. We prove this conjecture for $n \geq 425$. The condition that $n > r$ cannot be weakened.

Keywords: hypergraphs, set system, subgraph, regular graph.

2010 Mathematics Subject Classification: 05C65.

REFERENCES

- [1] N. Alon, S. Friedland, and G. Kalai, *Regular subgraphs of almost regular graphs*, J. Combin. Theory (B) **37** (1984) 79–91.
doi:10.1016/0095-8956(84)90047-9

¹Research of this author is partially supported by the Arnold O. Beckman Research Award of the University of Illinois at Urbana-Champaign.

²Research of this author is supported in part by NSF grant DMS-0965587 and by the Ministry of education and science of the Russian Federation (Contract no. 14.740.11.0868).

- [2] M. Kano, *Regular subgraphs of a regular graph*, Annals of the New York Academy of Sciences **576** (1989) 281–284.
doi:10.1111/j.1749-6632.1989.tb16409.x
- [3] D. Mubayi and J. Verstraëte, *Two-regular subgraphs of hypergraphs*, J. Combin. Theory (B) **99** (2009) 643–655.
doi:10.1016/j.jctb.2008.10.005
- [4] L. Pyber, *Regular subgraphs of dense graphs*, Combinatorica **5** (1985) 347–349.
doi:10.1007/BF02579250
- [5] L. Pyber, V. Rödl, and E. Szemerédi, *Dense graphs without 3-regular subgraphs*, J. Combin. Theory (B) **63** (1995) 41–54.
doi:10.1006/jctb.1995.1004
- [6] V. Rödl and B. Wysocka, *Note on regular subgraphs*, J. Graph Theory **24** (1997) 139–154.
doi:10.1002/(SICI)1097-0118(199702)24:2<139::AID-JGT2>3.0.CO;2-R
- [7] V.A. Tashkinov, *3-regular subgraphs of 4-regular graphs*, Mat. Zametki **36** (1984) 239–259..
- [8] V.A. Tashkinov, *Regular parts of regular pseudographs*, Mat. Zametki **43** (1988)) 263–275.

Received 20 June 2012
 Revised 30 January 2013
 Accepted 31 January 2013