

ON THE COMPLEXITY OF THE 3-KERNEL PROBLEM IN SOME CLASSES OF DIGRAPHS¹

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Abstract

Let D be a digraph with the vertex set $V(D)$ and the arc set $A(D)$. A subset N of $V(D)$ is k -independent if for every pair of vertices $u, v \in N$, we have $d(u, v), d(v, u) \geq k$; it is l -absorbent if for every $u \in V(D) - N$ there exists $v \in N$ such that $d(u, v) \leq l$. A k -kernel of D is a k -independent and $(k - 1)$ -absorbent subset of $V(D)$. A 2-kernel is called a *kernel*.

It is known that the problem of determining whether a digraph has a kernel (“the kernel problem”) is NP-complete, even in quite restricted families of digraphs. In this paper we analyze the computational complexity of the corresponding 3-kernel problem, restricted to three natural families of digraphs.

As a consequence of one of our main results we prove that the kernel problem remains NP-complete when restricted to 3-colorable digraphs.

Keywords: kernel, 3-kernel, NP-completeness, multipartite tournament, cyclically 3-partite digraphs, k -quasi-transitive digraph.

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