

A CONSTRUCTIVE CHARACTERIZATION OF VERTEX COVER ROMAN TREES

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Abstract

A Roman dominating function on a graph $G = (V(G), E(G))$ is a function $f : V(G) \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The Roman dominating function f is an outer-independent Roman dominating function on G if the set of vertices labeled with zero under f is an independent set. The outer-independent Roman domination number $\gamma_{oiR}(G)$ is the minimum weight $w(f) = \sum_{v \in V(G)} f(v)$ of any outer-independent Roman dominating function f of G . A vertex cover of a graph G is a set of vertices that covers all the edges of G . The minimum cardinality of a vertex cover is denoted by $\alpha(G)$. A graph G is a vertex cover Roman graph

if $\gamma_{oiR}(G) = 2\alpha(G)$. A constructive characterization of the vertex cover Roman trees is given in this article.

Keywords: Roman domination, outer-independent Roman domination, vertex cover, vertex independence, trees.

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