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METHODS OF MEASURING AND CALCULATING DISPLAY TRANSFER CHARACTERISTICS (GAMMA)

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Summary

It has always been found difficult to measure the transfer characteristic of a cathode ray display tube with sufficient precision, because of the problem of accurately setting the black level. To obviate this, a mathematical method has been devised which avoids this problem by calculating the black level error from the measured data. Using the same mathematical process it is also possible to calculate the uncontrolled, or stray, light which was present during the measurement process. In this way it is possible to derive the characteristic of the display as a power law (if one exists) with considerable precision.

The mathematics involved require computer processing of the data and an optimisation routine to provide ever-increasingly accurate estimation of the offsets in the data; however all the processing can be done on a small microcomputer using simple algorithms. Special routines have been developed for this and are listed in the Report.

Index terms: *CRT; gamma; measurement*

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1. INTRODUCTION

The function of a television system is to reproduce, as accurately as possible within given constraints, the appearance of an original scene at the eye of the observer. The constraints include the limited colour gamut and contrast range of the display, the accuracy of the camera and studio signal processing, distortions due to signal transmission, and the overall system transfer characteristic. The parameters of domestic displays are not directly under the control of the broadcaster, so a reference display is adopted to which domestic displays are assumed to approximate.

It is relatively easy to measure the colour gamut of a display[1], but it is much less so to measure the transfer characteristic which relates its light output to the drive voltage at the input. However, knowledge of that characteristic is required for the calculation of, for example, the colour correction matrices of studio cameras[2]. For simplicity, it is generally assumed that the transfer characteristic of the reference display takes the form:

$$L_d = k_d V^{\gamma} \qquad (1)$$

where V is the drive voltage, L_d is the resultant light output, and k_d is a scalar. The value of gamma is taken to be 2.8 for conventional cathode ray tube displays[3]. In order to compensate for this 'gamma' characteristic of the display, the camera is given an inverse characteristic such that the overall system is nominally linear. In practice, the actual value used is adjusted to allow for the fact that the display usually has a darkened surround. This modifies the perception of colours and is compensated by using an overall characteristic of greater than unity. Conventionally an overall value of 1.26 is assumed which, when combined with the display gamma of 2.8, yields a camera correction characteristic of 0.45[4]. Thus for the camera:

$$V = k_c L_c^{0.45} \qquad (2)$$

where L_c is the input light to the camera, and for the display:

$$L_d = k_d V^{2.8} = k_s L_c^{1.26} \qquad (3)$$

$$k_s = k_d k_c^{2.8}$$

However, for practical reasons, the story is not as simple as this. This idealised camera characteristic has a slope (gain) of infinity near black, and thus camera noise would be heavily emphasised. As a compromise, the practical camera characteristic is adapted so that it has a linear section near black with a maximum gain of, say, 5; the remainder of the curve follows a modified power law.

Thus a knowledge of the camera and display transfer characteristics is vital for the full understanding of any television system. Measurement of the display transfer characteristic is made difficult by the presence of black level setting and other offsets, each of which makes the extraction of the power law from the measured data rather unreliable. Consequently, knowledge of the overall performance of television systems is only approximate. Any improvement in the extraction of the power law from the data would automatically improve the accuracy of calculations on television systems.

This Report describes a new method of extraction of the display power law from measured data, but first describes traditional methods and their limitations as they apply to measurement of cathode ray tube displays. It should be noted that the techniques described can be applied equally to camera transfer characteristics, or to any other set of data which may contain a power law relationship.

2. BASIC MEASUREMENT TECHNIQUE

The test signal used for assessment of display gamma is typically a white patch which may be moved to the area of the display under investigation. In practice it is rarely necessary to move the patch since there is no evidence that the transfer characteristic varies over the display area.

The light from the display is measured using a photometer, preferably with digital output, which must respond accurately over many decades of light level. The photometer must have good linearity, since its excitation will be heavily modulated at both field and line scan rates; a photomultiplier is a suitable device since it has many decades of noise-free linear operation. The photometer can be in contact with the display or relay optics can be used to make remote measurements; either method is suitable.

The transfer characteristic can then be measured by successively attenuating the video signal and measuring the light level from the display together with the driving voltage. Thus a table of voltage and light levels is obtained. Fig. 1 shows the results of three such measurements, made with three different black level settings, for the green channel of a real cathode ray tube display; no attempt was made to eliminate measurement offsets. The three sets of values were recorded, first with the display brightness set too low, then approximately correct, and then set too high. All measurements were made in total darkness. The data values are listed in Table 1 as *L*− *L* and *L*+. They are typical of cathode ray tubes but do not directly yield a value for gamma. The values are given in arbitrary units and refer to a white level setting of about 80 cd/m^2. Results were not recorded for greater than 30 dB attenuation in the 'sat-down' case since they showed no further change in light level.

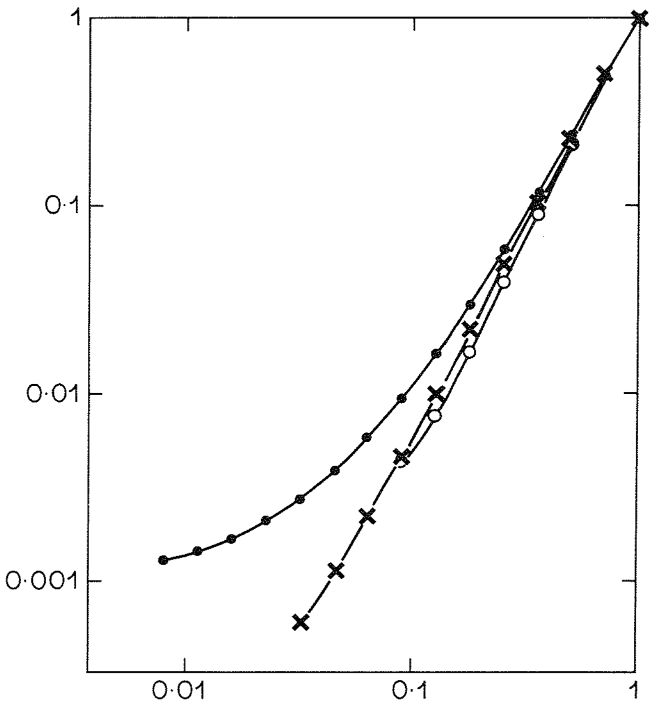

Fig. 1 - Normalised plot of three measurements of a CRT display.

- • *L*+ 'sat up'
- × *L* 'correct sit'
- o *L*− 'sat down'

It should be noted that the light output of a cathode ray tube display depends not only on the video drive voltage, but also on the clamp circuitry, scan amplitudes and electron accelerating voltages. In a broadcast quality monitor such as was used for the measurements listed above, these should all be temporally invariant but some projection displays and most domestic receivers may not be sufficiently stable. For these it may be necessary to make frequent readjustments of the black level and under these conditions the data values will be less reliable.

Table 1

dB	*L*−	*L*	*L*+
−42		0.008	0.137
−39		0.012	0.154
−36		0.018	0.181
−33		0.028	0.221
−30	0.230	0.055	0.288
−27	0.240	0.102	0.409
−24	0.272	0.199	0.614
−21	0.364	0.413	1.00
−18	0.618	0.870	1.73
−15	1.32	1.91	3.16
−12	3.07	4.20	6.07
− 9	7.24	9.19	12.0
− 6	17.0	20.2	24.4
− 3	38.6	43.7	50.0
0	86.5	94.8	104.0

Traditionally, the display black level can be set using the PLUGE test signal[5] which has two bars near black, one at +3% which must be visible, the other at −3% which must be invisible. Thus the black level can be set to only 3% accuracy. This is inadequate for high precision measurements but unfortunately there is no real alternative test signal available.

3. GAMMA EXTRACTION

3.1 Graphical method

The traditional approach is to plot the measured values on logarithmic graph paper and to draw a linear-regression line through the points. The slope of this line is then gamma since:

if $$L_d = k_d \, V^{\gamma} \qquad (1)$$

then $$\log (L_d) = \log(k_d) + \gamma \log(V) \qquad (4)$$

The line can be drawn either by hand, with visual inspection of the data points to establish the slope, or by a linear-regression analysis computer program such as is listed in Appendix 1. Unfortunately there is no single straight line which will satisfactorily run through all the data points in the logarithmic plots of Fig. 1 since there is considerable curvature on them. This curvature results directly from errors in the setting of black level and the 'zero light' response of the photometer. Normally the straight line is drawn only through the points starting from the highest signal level, in which the errors have least effect. Fig. 2

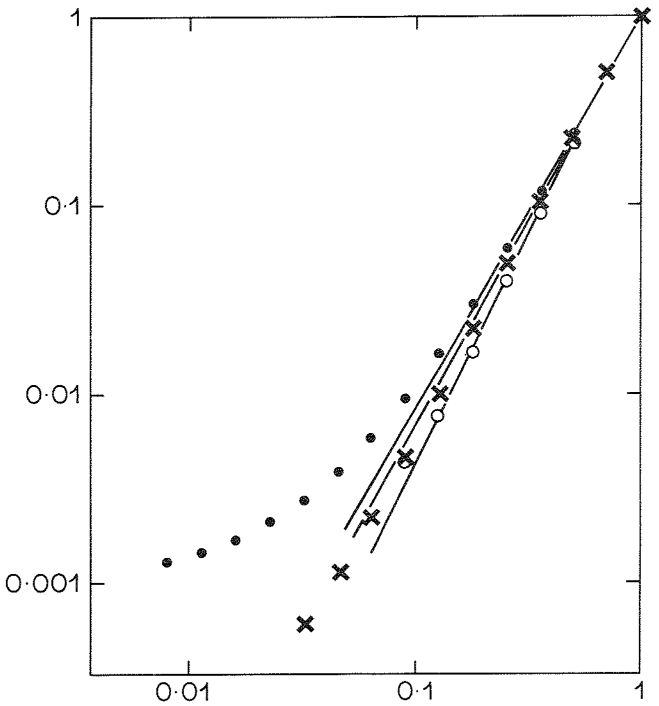

Fig. 2 - Normalised plot of three CRT measurements with trend lines at high brightness only.

- • $L+$ 'sat up'
- × L 'correct sit'
- ○ $L-$ 'sat down'

shows regression lines drawn through the top four points of each line in Fig. 1, their slopes are 2.39198, 2.25135 and 2.08342. Unfortunately, this process ignores many of the measurement points and does not produce a consistent value for gamma; indeed, it might be concluded from this that the value of gamma changes with both brightness and signal level, but this would be erroneous, since the physical characteristics of the CRT cannot be dependent on the incoming signal.

It is possible to improve the accuracy by using greater care in the measurement process, principally in the setting of black level and by measuring and subtracting the zero light signal from the photometer. There is much scope for error; particularly since the PLUGE signal only offers 3% adjustment limits, it is pointless trying to achieve greater accuracy than this.

This method is generally unsatisfactory as it requires great skill on the part of the operator and is frequently unrepeatable. Clearly, a better method is needed.

3.2 Manual numerical method

One possibility for an improved analysis of the raw measurements is to try and derive, by inspection of the data or other means, values for the black level and zero light signals. The transfer characteristic equation (1) is redefined so as to include the effects of these undesired effects:

$$L_d + l_o = k_d(V + v_o)^{\gamma} \qquad (5)$$

where k_d is still a scaling constant. The constant l_o is the combined effect of dark current in the photometer, stray light, and the 'zero-drive' light from the display. The voltage constant v_o is the drive voltage which produced the zero-drive light, and is the error in black level setting.

The precision of voltage values can be improved in the measurement process. If signal voltages are recorded using an oscilloscope or video meter, then errors will probably be present. If, however, a calibrated attenuator is used, the nominal drive voltage can be calculated from its settings to the accuracy of the attenuator calibration. Thus for an ideal attenuator:

$$V = V_{max} \,.\, 10^{\left(\frac{dB}{20}\right)} \qquad (6)$$

where V_{max} is the peak signal voltage, and the precision of the values is a function of this calculation process only. This process was used for all calculations described in this Report. The value of v_o occurs within the display itself, and thus is unaffected by any attenuator settings.

The light offset l_o can be significantly reduced by accurately measuring the zero-light level when very large attenuation is applied to the test signal. This procedure was adopted for the measurements made with black level set nominally correctly and for those with black level set high.

Neither of these practices eliminates the offsets entirely, so a method which takes better account of the offsets in the measured data is to be preferred.

3.3 Automatic numerical method

If some means could be found of estimating both l_o and v_o from the measurement data values, the true value of gamma could be found using the graphical method described above. Fortunately, computers provide more than adequate power to enable the necessary series of optimisations to be carried out, as will be shown in the remainder of this section.

Fig. 3 shows a logarithmic plot of a true power law over two decades of horizontal (X) values, but with an offset added to the vertical ordinate, ie:

$$Y = y_o + k\,X^{\gamma} \qquad (7)$$

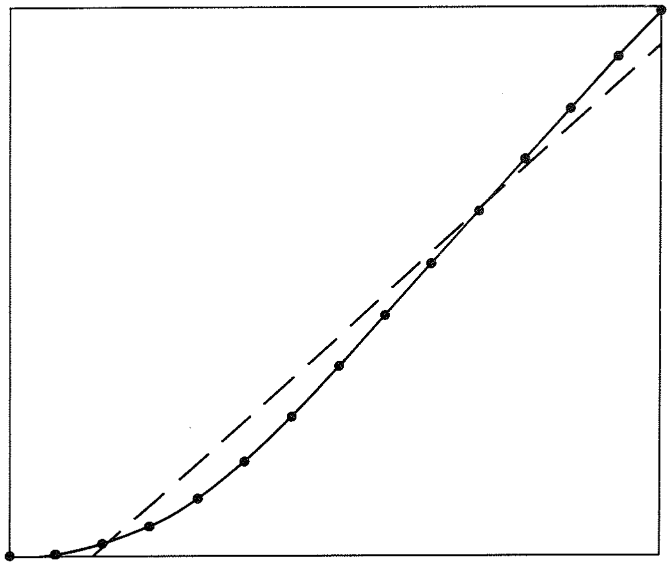

Fig. 3 - Logarithmic plot of $Y = 0.0001 + X^{2.5}$.

The value for y_o is only 0.01% and gamma is 2.5. The gamma value found from the regression line through all of the data points is 2.0275 and there is curvature in the data points only at the lower end of the curve. A regression analysis for the top two points produces a gamma value of 2.4996, which is very close, but not identical, to the actual value of 2.5.

Similarly, Fig. 4 shows the same power law plotted with an offset in the horizontal ordinate, ie:

$$Y = k(X + x_o)^{\gamma} \qquad (8)$$

The value for x_o is 1% and gamma is still 2.5. The value of gamma found from all the data points is 2.9082 and there is data curvature over the entire range. The slope of the line through the top two points is 2.5299, close to but not exactly 2.5.

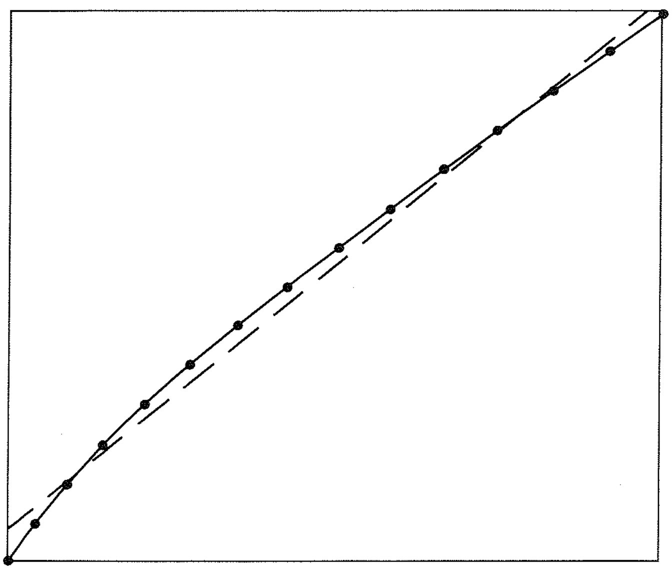

Fig. 4 - Logarithmic plot of $Y = (X + 0.01)^{2.5}$.

Armed with this knowledge of how offsets affect curvature, a skilled operator can estimate the values of offsets contained in the data for a real display, such as is listed in Table 1, to produce a better approximation to the true gamma, but it is very difficult and time consuming to separate the effects of the two offsets. In return for the skill of interpretation, the setting of black level is now largely irrelevant; it need only be set such that the display is not cut off at the lowest measurement point. Although the regression line analysis will now use all the data points, the process is time consuming and requires a considerable degree of skill. An automated method is to be preferred.

To test this, the data values for the monitor, listed in Table 1, were reprocessed using calculated values for V derived from the attenuator settings, and the set of light readings derived for the monitor when black level was nominally correct. Initially, it was assumed that v_o was zero and the value of l_o was estimated by using an optimisation routine which adjusted l_o until the curvature on the data points about the linear-regression line was minimised. Since only one variable was to be optimised, a simple algorithm could be used to determine it with great precision.

Clearly, in order to carry out this optimisation a mathematical measure is required for the curvature of the data points about the regression line. The one used for this work operated by summing the differences between each actual data value and the calculated value, over each quartile of the measurement range. The two central quartile subtotals were added together and the two outer quartile subtotals were both subtracted thus producing a signed assessment of curvature. The operation is shown diagrammatically in Fig. 5 and a computer routine which does this is listed in Appendix 2. This measurement is sensitive to simple curvature only; higher order curvature such as 'S' shapes are misinterpreted. Undoubtedly, a better measure of curvature could be devised, but the one described was entirely sufficient for the work. Appendix 3 shows the results of using these computer routines to analyse some test data with known offsets and gamma.

The result obtained for the monitor measurements made with nominally correct black level was:

$$L_d = 0.01715 + 95.41\ V^{2.26643} \qquad (9)$$

Analysis was done over the data range from 0 dB to −30 dB, as values at higher attenuation were suspect because the photometer was operating near to

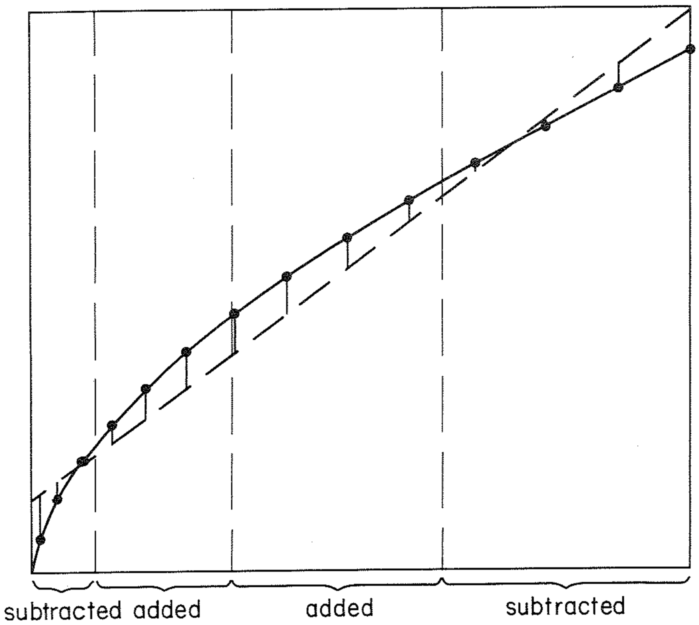

Fig. 5 - Curvature assessment, outer quartiles are subtracted from inner quartiles, preserving signs.

its lower limit. The result is shown in Fig. 6. The data curvature was $1.57*10^{-11}$ which is very small. This indicates that it was reasonable to ignore the effect of v_o for this data set.

It is perfectly feasible to assume, instead, that l_o is zero, and to set the computer algorithm the task of finding a value for v_o. However, it is not possible to find a value for either offset in the presence of the other, since both offsets give rise to data curvature; thus this process cannot be carried out on either of the two sets of data from Table 1 in which the black level was incorrectly set. To illustrate this the same process was adopted, that of assuming v_o is zero and finding a value for l_o, on the other two sets of data:

sat down

$$L_d = -0.08537 + 87.287\ V^{2.42492} \qquad (10)$$

sat up

$$L_d = 0.13338 + 100.11\ V^{1.94496} \qquad (11)$$

The full range of data values was used for the 'sat-up' black level, only data in the range 0 dB to −21 dB were used in the 'sat-down' case for the reasons described above. The results are shown in Figs. 7 and 8 which both exhibit considerable

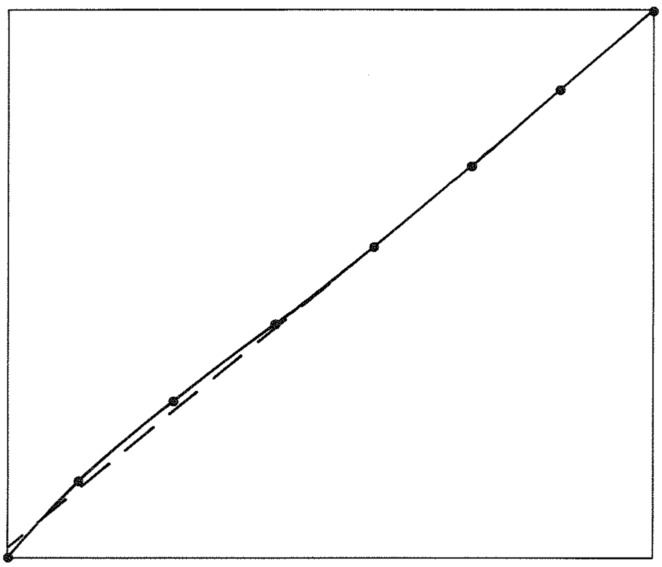

Fig. 7 - Normalised plot for CRT 'sat up', incorporating optimised value for l_o and ignoring v_o.

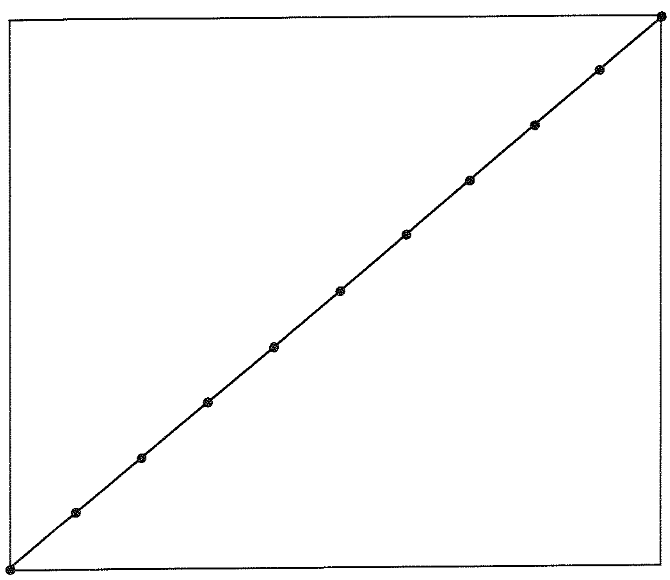

Fig. 6 - Normalised plot for CRT with 'correct sit', incorporating optimised value for l_d and ignoring v_o.

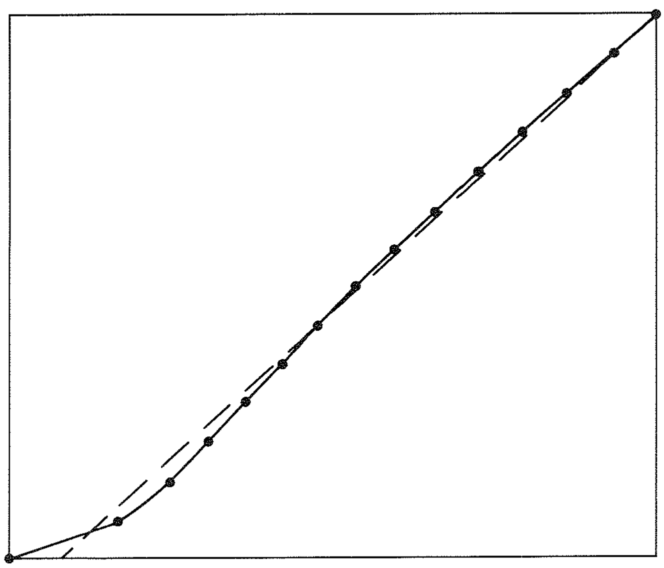

Fig. 8 - Normalised plot for CRT 'sat down', incorporating optimised value for l_o and ignoring v_o.

curvature, thus indicating that it was not safe to ignore the effects of v_o. Also, the values of gamma differ from each other and from the value found in equation (9) for the data set with nominally correct black level.

Clearly, if both l_o and v_o could be found independently of each other a more consistent value could be found for gamma.

3.4 Differential method

In order to analyse measurements taken on monitors under sat up or sat down conditions, a more sophisticated version of the numerical analysis given in Section 3.3 was devised; this makes it possible to eliminate either of the two offsets from the data by purely mathematical means, involving no skill on behalf of the operator. Recalling the relationship of equation (5) which contains the assumed offsets, it is possible to differentiate this[6] to obtain:

$$\frac{dL_d}{dV} = k_d \, \gamma \, (V + v_o)^{(\gamma-1)} \qquad (12)$$

from which the value of l_o has vanished. Thus if the slope of the original curve (directly measurable from Fig. 1) is plotted versus the drive voltage (both logarithmically) then the regression line through the data points has a slope whose value is $\gamma-1$, or:

$$\gamma = 1 + slope \qquad (13)$$

The value of voltage offset (the drive voltage which produced the value of zero light) is still present, but computer optimisation can adjust a value for v_o so as to reduce the curvature of the data points about the regression line as was described in the previous section.

The data values for dL_d/dV are obtained as the ratio of differences in adjacent values of the original data for L_d and V, thus for sample n:

$$\frac{dL_d}{dV} = \frac{[(L_d(n) + l_o) - (L_d(n+1) + l_o)]}{[(V(n) + v_o) - (V(n+1) + v_o)]}$$

$$= \frac{[L_d(n) - L_d(n+1)]}{[V_d(n) - V_d(n+1)]} \qquad (14)$$

from which it can be seen that neither of the offsets, l_o and v_o, enters into the calculation of dL_d/dV provided that neither offset varies from sample to sample. The values of dL_d/dV for the central set of measured data listed in Table 1 are given in Table 2. The values of V and L_d listed here are the arithmetic means of adjacent pairs of the original values from Table 1 and are used as the voltage or light level at which the calculated slope is assumed to be effective.

Table 2

dL_d/dV	V	L_d
0.00360	0.03815	0.00008
0.00526	0.05388	0.00015
0.00822	0.07611	0.00031
0.01243	0.10751	0.00064
0.02002	0.15186	0.00139
0.03122	0.21451	0.00306
0.04806	0.30300	0.00669
0.07529	0.42800	0.01469
0.11366	0.60457	0.03195
0.17497	0.85397	0.06925

Analysis of the data listed in Table 2 for dL_d/dV versus V, without attempting to optimise a value for v_o, produced the following equation:

$$L_d = l_o + k_d V^{2.26386} \qquad (15)$$

where l_o is unknown. The curvature of $1.17*10^{-7}$ is small, indicating that v_o is also small. No value for k_d was produced at this time. An attempt to find a value for v_o by minimising the curvature produced:

$$L_d = l_o + k_d(V - 0.00003129)^{2.26386} \qquad (16)$$

The optimised curvature was $1.04*10^{-7}$ and the value of v_o is only 0.003%, and again no value was produced for k_d. For this process, the slope dL_d/dV was notionally plotted versus the mean voltage of the two adjacent values from which dL_d/dV was calculated, thus for sample number n:

$$X(n) = \frac{(V(n) + V(n+1))}{2} \qquad (17)$$

This may not be the optimum calculation for the horizontal axis but it produces acceptable results.

It is also possible to eliminate v_o from the calculation by substituting for $V + v_o$ in equation (12) from equation (5):

$$\frac{dL_d}{dV} = k_d \, \gamma \left(\frac{L_d + l_o}{k_d} \right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \frac{k_d \, \gamma}{k_d^{\left(1-\frac{1}{\gamma}\right)}} \, (L_d + l_o)^{\left(1-\frac{1}{\gamma}\right)} \qquad (18)$$

This time, plotting the slope dL_d/dV versus L_d logarithmically produces data whose slope is

1−1/gamma; thus for this plot the horizontal axis is the mean light level of adjacent values:

$$X(n) = \frac{(L_d(n) + L_d(n+1))}{2} \qquad (19)$$

and

$$\gamma = \frac{1}{(1 - \text{slope})} \qquad (20)$$

Again, a computer is used to optimise the value of l_o such that the data points do not curve about the linear regression line. Analysis of the values in Table 2, the central set of values from Table 1, plotting dL_d/dV versus L_d, produced the following equation:

$$L_d = -0.0185839 + k_d(V+v_o)^{2.25397} \qquad (21)$$

with a curvature of $-3.38*10^{-13}$. Again, no attempt was made to find a value for k_d.

The optimised values of gamma from equations (16 and 21) are extremely close to each other. As a check on the two differential processes, the optimised values of l_o and v_o respectively were both subtracted from the original data and a new linear regression line calculated for this processed data using the conventional L_d versus V analysis as described in section 3.1. Fig. 9 shows the result, and the equation found is:

$$L = -0.018584 + 96.0383(V-0.00003129)^{2.27353} \qquad (22)$$

with a data curvature of $1.53*10^{-7}$.

The same processes were carried out on the two other sets of data for the monitor and all the results are listed in Table 3.

In this table, values in parentheses are those assumed in the calculation, N/A indicates that a value was not relevant.

Clearly, the value for gamma lies between 2.20341 and 2.27353 if the simple analysis results are discounted. This range of gamma values is only 3% and the standard deviation of the group is 1.14%, which confirms that the methods are mathematically sound. The different curvatures found in each analysis cannot be directly related to each other since they each refer to different scales; they can only be used to compare results obtained for similar analyses (L_d vs V, dL_d/dV vs L_d, or dL_d/dV vs V) and over the same range of data values.

It seems appropriate to take, as the gamma value for each setting of black level, the mean of the three values found by the optimisation process (but see the discussion below); these means are 2.22530, 2.26369 and 2.22758 for the three black level settings respectively. The results show no evidence to support any assumption that the underlying value of gamma changes with black level setting and thus the true value for that display can be taken as the mean of these three means, or 2.2388.

All of the results given in Table 3, together with these calculated means, are shown in Fig. 10.

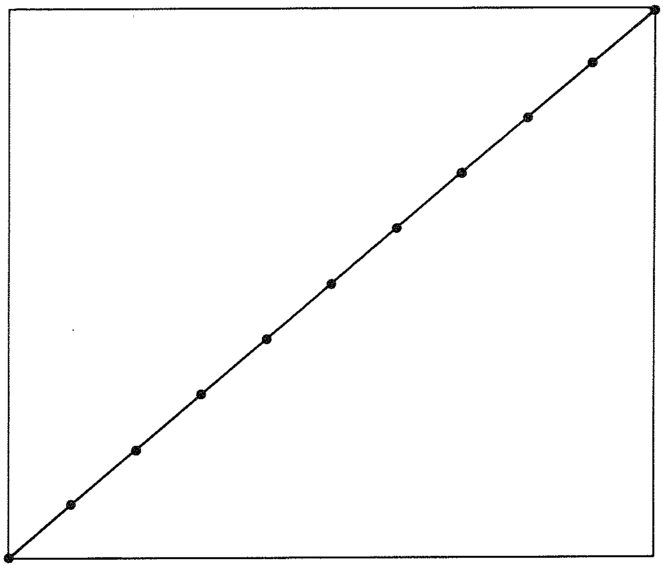

Fig. 9 - Normalised plot for CRT 'correctly sat' incorporating optimised values for l_o and v_o.

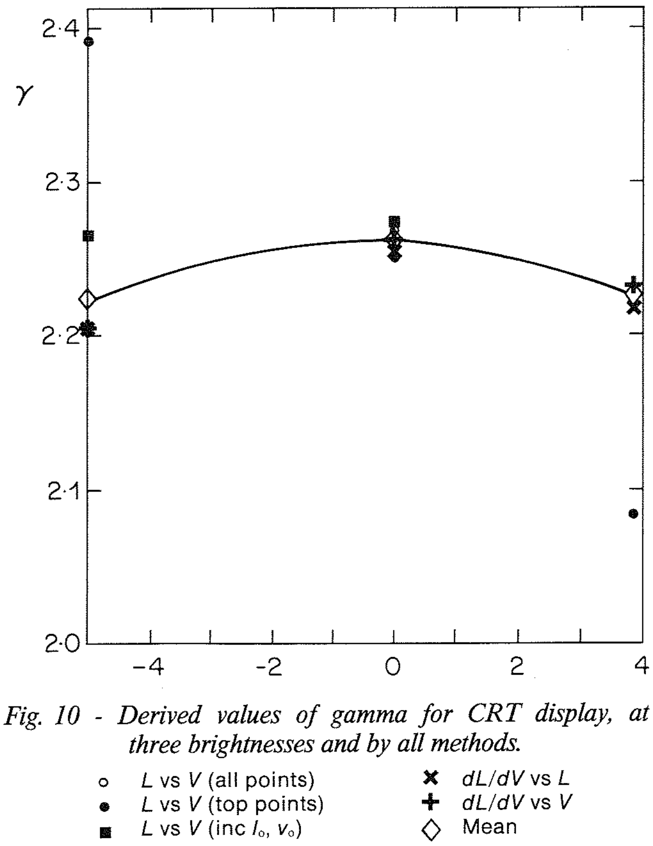

Fig. 10 - Derived values of gamma for CRT display, at three brightnesses and by all methods.

Table 3

Results for real monitor (sat down)

Method	γ	l_o	v_o
L vs V (all points)	2.42492	(0)	(0)
L vs V (top points)	2.39198	(0)	(0)
dL/dV vs L	2.20409	−0.0363745	(N/A)
dL/dV vs V	2.20341	(N/A)	−0.0501234
L vs V (inc l_o, v_o)	2.26841	(−0.0363745)	(−0.0501234)

Results for real monitor (correctly sat)

Method	γ	l_o	v_o
L vs V (all points)	2.26643	(0)	(0)
L vs V (top points)	2.25135	(0)	(0)
dL/dV vs L	2.25397	−0.0185839	(N/A)
dL/dV vs V	2.26357	(N/A)	−0.0000313
L vs V (inc l_o, v_o)	2.27353	(−0.0185839)	(−0.0000313)

Results for real monitor (sat up)

Method	γ	l_o	v_o
L vs V (all points)	1.94496	(0)	(0)
L vs V (top points)	2.08342	(0)	(0)
dL/dV vs L	2.21963	−0.0342441	(N/A)
dL/dV vs V	2.23266	(N/A)	0.0386334
L vs V (inc l_o, v_o)	2.23044	(−0.0342441)	(0.0386334)

4. DISCUSSION

It is not surprising that the various values of gamma are not all identical, there are several possible causes for this:

1. The measurement data values were recorded only to three significant figures; this was because power supply hum-modulation caused slow changes in light level. Even very low levels of power supply variation cause measurable changes in light level; if the test signal is not phase-locked to the monitor supply power, small cyclic variations in light level will be observed. It is difficult to make precise measurements under these conditions.

2. The optimised values of offsets may be close to the lowermost measured values; thus the lower values may be relatively unreliable since only one or two digits are actually relevant. Appendix 3 shows the results of analysis on artificial data recorded to various numbers of significant figures. Nevertheless, the results show remarkable consistency.

3. The precision of the value of the optimised offsets is controlled by the accuracy and precision of the measured data. This is particularly relevant when slope-related calculations are made, since ratios are taken of small differences. Thus the results of the differential process (dL_d/dV versus L_d or V_d) must be inherently less accurate than those for the direct process (L_d versus V).

4. The optimised values for l_o and v_o are not absolutely precise, being derived from a

statistical process, and thus there may be residual errors remaining in the data. This causes data curvature, however small, and results in errors in the values of gamma.

5. It was assumed that the slope between two points on the original curve (Fig. 1) is effective at the mean value of L_d or V; this assumption is not strictly valid for finite intervals where a power law is involved. It might be better to take, for example, as a mean voltage, the value derived from the mean attenuator setting in dB. It may be better to measure with much smaller intervals of drive voltage V; these need not be contiguous since measurements can be made in pairs at, say, 0.1 dB intervals, each pair centred on drive signals at, say, 3 dB apart. Thus measurements would be at 0 −0.1 −3 −3.1 −6 −6.1 etc. However, such measurements would have to be made with great precision to ensure data reliability since the differences taken in the slope calculations would be very small.

Having established the offsets (l_o and v_o where applicable) by the differential process, there is no mathematical way in which reiterating the processes could produce more accurate results since each offset is calculated totally independently of the other.

There may be a philosophical problem with this data manipulation. Is it permissible to add or subtract constants to measured data values? This problem resolves into a different question; do we wish to know the transfer characteristic in the form of a look-up table for the device as it was tested, or do we require the underlying power law equation? The actual characteristic of the display is the voltage-to-light curve such as is shown in Fig. 1, but that is of little use in making overall system performance calculations since it contains unmeasured offsets which may not be repeatable; under these circumstances a power law would be much more useful. Even if the offsets are genuine and repeatable, values for them are required in order to characterise the system by an equation. Since the offsets in both voltage and light have physical significance (they are both independently measurable, although with great difficulty), a mathematical process which reveals them can be used to extract the pure power law if one exists. It is easy to set limits on the offsets, the voltage offset should not be greater than 3%, since the black level can be set to better than this, and the light offset will always be positive and smaller than the smallest measured value.

If a true power law has been found by these processes, then the performance of the display under any conditions can be calculated by adding constants which model the practical circumstances. Thus the display can be modelled by:

$$L = l_o + k(V + v_o)^{\gamma} \qquad (23)$$

which is no longer a true power law.

Finally it should be noted that if neither a skilled operator nor an optimisation routine can find values of offset which reveal a true power law, then none exists and the display cannot be modelled in this fashion. This is likely to be true for some new display devices, not based on cathode ray emissions, such as lasers and liquid crystals.

5. CONCLUSION

New methods have been described, in which the power law connecting two sets of data can be extracted in the presence of unwanted additive constants. Either a trained operator or a computer optimisation routine can be used to establish the power law and constant offsets. This largely eliminates the need for accurate setting of black level when measuring a cathode ray tube display. Using this method it is possible to extract the power law (gamma) of a display device with considerable precision.

6. ACKNOWLEDGEMENTS

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APPENDIX 1

A Routine for Least-Square-Errors Regression Analysis

This routine is written in BBC BASIC as a procedure, operating on two data arrays, 'x( )' and 'y( )', in which there are 'N' values. It is required that data values in the x( ) array increase monotonically as the array index increases. The arrays may contain either the actual data values for light and voltage, or the slope dL/dV and light or voltage as required. The offset value being optimised is 'off'.

The routine finds values of 'slope' and 'intercept' which define the regression line. The usefulness of the line is assessed in 'reliability'.

NOTE: in BBC BASIC LN denotes taking natural logarithm, SQR denotes the taking of a square root, and x^2 denotes raising x to the power 2.

```
1000 DEFPROCregress
1010 xsum=0:ysum=0:xy=0
1020 xsum2=0:ysum2=0
1030 FOR I=0 TO N
1040   x=x(I)+off:y=y(I)
1040   IF x>0 x=LN(x) ELSE x=0
1050   IF y>0 y=LN(y) ELSE y=0
1060   xsum=xsum+x:ysum=ysum+y
1070   xy=xy+x*y
1080   xsum2=xsum2+x^2:ysum2=ysum2+y^2
1090 NEXT
1100 Y=xy-xsum*ysum/(N+1)
1110 X=xsum2-xsum^2/(N+1)
1120 slope=Y/X
1130 intercept=(ysum-slope*xsum)/(N+1)
1140 reliability=SQR(Y*slope/(ysum2-ysum^2/(N+1)))
1150 ENDPROC
```

APPENDIX 2

A Routine for Data Curvature Assessment

First a value of 'y' is calculated using the values of 'intercept' and 'slope' derived from PROCregress. Then it is accumulated into the variable 'curl' as the square of the difference between y and the data value in the y( ) array, taking note of the sign of the difference, which is reversed if the data point lies in either of the two outer quartiles of the data range.

Finally the variable 'curl' is normalised to the number of values and the maximum value occurring. This is strictly unnecessary and is only included so that curvatures can be compared between different data sets. Note that the curvature for a LOGL vs LOGV plot is not directly comparable with that for a LOGdL/dV vs LOGL or LOGV plot. The reliability of curl increases with the number of data points. The routine exits by returning the value of curvature.

NOTE: in BBC BASIC SGN denotes taking the sign of its argument, it returns values −1 when negative, +1 when positive, 0 when zero.

```
2000  DEFFNcurl
2010  curl=0
2020  FOR I=0 TO N
2030  y=(EXP(intercept))*(x(I)+off)^slope
2040  IF I<N/4 OR I>3*N/4 sign=-1 ELSE sign=1
2050  curl=curl+sign*SGN(y(I)-y)*(y(I)-y)^2
2060  NEXT
2070  =curl/(N+1)/x(N)
```

APPENDIX 3

Test Calculations on Data Containing Known Offsets

A set of data was generated from the formula:

$$L = 0.000987654321 + (V + 0.0123456789)^{2.5}$$

over a 42 dB range in 3 dB steps, and the computer routines described in Appendices 1 and 2 were used within an optimisation routine to find the values of the offsets and gamma.

Fig. A3.1 shows the data plotted logarithmically, with a regression line calculated over the whole data. Its slope (gamma) is 1.8299 which is clearly wrong. Fig. A3.2 shows the same data plot with a regression line calculated over only the top two points. Its slope is 2.5329, which is still in error by more than 1%.

Optimised logarithmic regression analysis of dL/dV versus L found a value of l_o by minimising the curvature of the points. The equation produced is:

$$L = 0.0009876543222 + V^{2.499999981}$$

with reliability of 0.9999999998 and curvature of $-8.658*10^{-18}$. Similarly for dL/dV versus V, v_o was optimised and the equation is:

$$L = (V + 0.01234567878)^{2.500000005}$$

with reliability of 0.9999999993 and curvature of $2.303*10^{-19}$. Fig. A3.3 shows a logarithmic plot, similar to Fig. A3.1, but incorporating these values of l_o and v_o. The equation of the regression line is:

$$L = 0.0009876543222 + (V + 0.01234567878)^{2.500000004}$$

with a reliability of 1 and curvature of $-7.336*10^{-21}$.

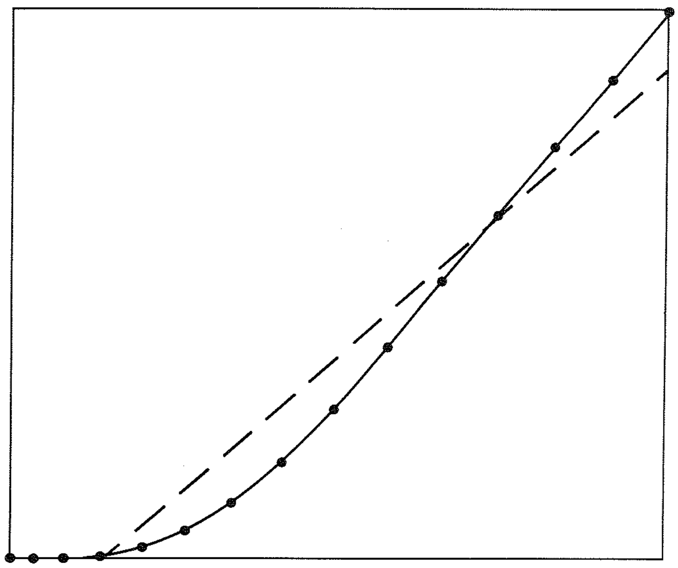

Fig. A3.1 - Logarithmic plot for $L = V^{2.5}$ with offsets as described in the text. Regression line calculated for all data points.

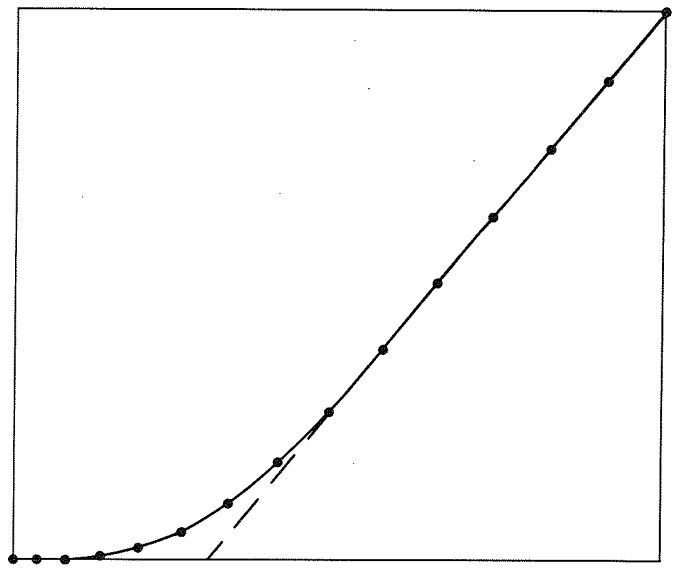

Fig. A3.2 - Logarithmic plot for $L = V^{2.5}$ with offsets as described in the text. Regression line calculated for top two points only.

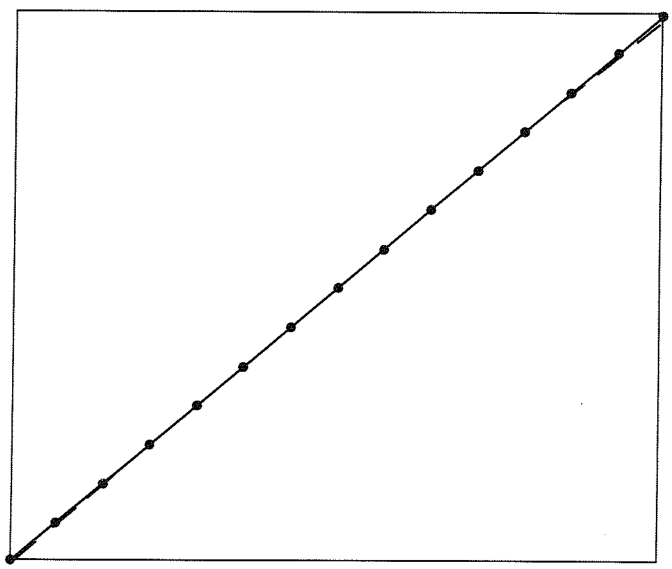

Fig. A3.3 - Logarithmic plot for $L = V^{2.5}$ with offsets as described in the text. Regression line calculated for all points, incorporating optimised offset values for l_o and v_o.

APPENDIX 4

The Effect of Limited Numbers of Digits in Measured Values

A data table was generated using the relationship:

$$L = V^{2.5}$$

over a range of V corresponding to attenuations of 0 to −42 dB, in 3 dB steps. The values of L and V were then each rounded to five then four then three then two digits before analysis. The results are given below:

digits	γ	reliability	curvature
2	2.497784042	0.9999617049	$-5.501*10^{-6}$
3	2.499466946	0.9999997779	$-1.093*10^{-7}$
4	2.500011800	0.9999999998	$-2.097*10^{-10}$
5	2.499997779	1.0000000000	$8.590*10^{-13}$

Clearly, the value of gamma found is very close even with only two significant figures in the data, and accuracy increases as the number of digits increases.

The same experiment was repeated, using 10-digit precision for V and variable rounding for L, with the following results:

digits	γ	reliability	curvature
2	2.501150972	0.9999951289	$5.740*10^{-7}$
3	2.500211701	0.999999962	$-4.206*10^{-9}$
4	2.499988625	0.9999999986	$-3.994*10^{-11}$
5	2.500001254	1.0000000000	$8.413*10^{-13}$

It is evident that if only L is rounded, the results are about 10 times more accurate.